16831 Statistical Techniques, Fall 2009: Homework 1

Due: Tuesday, September 15, beginning of the class

Instructions

There are 4 questions on this assignment (3 pages). These are short, simple problems. Included is the maximum answer length. If it is longer than this, it is likely wrong! (And certainly hard to read.)

1 Importance Sampling

1.1

Let $P(X)$ be a distribution, and $f : \mathbb{R} \to \mathbb{R}$ be a real valued function. In class, we discussed computing $\mathbb{E}_P[f(X)]$ using importance sampling, where one draws $X^{(1)}, \ldots, X^{(R)}$ from some importance distribution Q and estimates the expectation using:

$$\hat{\mu} = \frac{1}{R} \sum_{r=1}^{R} w^{(r)} f(x^{(r)}),$$

where $w^{(r)} = P(x^{(r)})/Q(x^{(r)})$.

The question is: for what choice of importance distribution Q, is the variance of $\hat{\mu}$ minimized? Assume that Q(x) is nonzero whenever P(x) is nonzero.

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2 Occupancy Maps

We discussed in class viewing occupancy map as a collection of 1D binary bayes filters.

2.1

a) What is the motion model in this filter?

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b) The simplest model of dynamics for this system allows the underlying state x_i of each map cell to transition with Markov dynamics between filled and empty. Consider what modifications that are necessary to the occupancy grid algorithm if the probability of transitioning between filled and empty is symmetric and happens with probability α. Point out the changes. Is this a reasonable model of dynamic obstacles?

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2.2

As discussed in class, there is a step in the derivation of the occupancy grid algorithm that is false for any reasonable model of sensors on robots. Go through the derivation of the binary bayes filter in the book and identify this step. I.e., we discussed that there is something fundamentally wrong with pretending mapping is running a lot of 1-dimensional filters. Where does this assumption not make sense in the derivation?

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